Field in Single-Mode Helically-Wound Optical Fibers

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Abstract —The scalar field wave equations of the fundamental mode in single-mode helically-wound optical fibers with circular cross section are obtained by using the Maxwell equations in the local orthogonal curvilinear coordinate system introduced by Tang. Two important results are brought about: 1) The field in the above-mentioned fibers maintains a quasi-linear state of polarization while its orientation rotates with a rotation rate close to $-\tau$ with respect to the Serret–Frenet frame. 2) The state of polarization (SOP) of the above field changes periodically along the propagation distances from 1 to a value a little less than 1, and, for a fixed s, it changes periodically according to the incident polarized angles with a period $\pi/2$. The theoretical results have been verified by the experimental measurements.

I. Introduction

FIELDS IN HELICALLY-WOUND optical fibers were studied experimentally by Papp and Harms [1]. Ross induced the experimental results to an empirical axiom [2]. However, reports concerning the solutions of the Maxwell equations in a helically-wound optical fiber have not been seen in the literature. A general form of the Maxwell equations in a helical system was first set up by Sollfrey [3]. It tends to be troublesome to treat the equations exactly due to its nonorthogonality. In differential geometry, the Serret–Frenet frame is generally used. Yet, this frame is also nonorthogonal insofar as the torsion is not equal to zero [4]. Based on this frame, Tang introduced a derived system—Tang's coordinate system [5]. It is a local curvilinear orthogonal system of space, in which the field equations in a helically-wound waveguide have a compact form for perturbation analysis. This paper discusses the Maxwell equations and the scalar field wave equations of helically-wound optical fibers in Tang's coordinate system. Two important results are brought about: 1) The field in the above fibers maintains a quasi-linear state of polarization while its orientation rotates with a rotation rate close to $-\tau$ with respect to the Serret–Frenet frame. 2) The state of polarization (SOP) of the above-mentioned field changes periodically along the propagation distance s from 1 to a value a little less than 1, and, for a fixed s, it changes periodically according to the incident polarized angles with a period $\pi/2$.

The above conclusions have been verified by the experiments of Papp and Harms [1].

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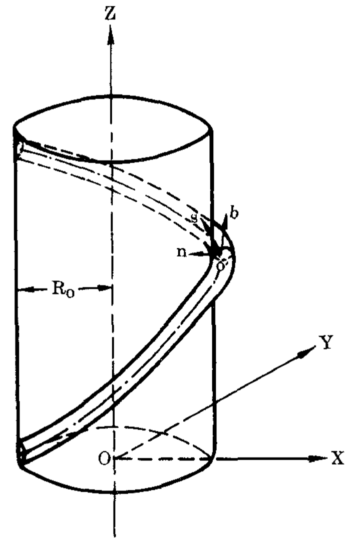

Fig. 1. Geometry and coordinate systems of a helically-wound optical fiber.

II. The Field Equations in Tang's Coordinate System and the Zero-Order Approximatic Solutions

Fig. 1 shows the geometry of a helically-wound optical fiber. R_0 is the radius of the cylinder and a the radius of the fiber core with

$$R_0 \gg a. \tag{1}$$

The center line of the fiber is a right circular helix, while $2\pi B$ and σ are its pitch and pitch angle. respectively, s is the arc length, χ and τ are the curvature and torsion, such that

$$\chi = R_0/(R_0^2 + B^2) \quad \tau = B/(R_0^2 + B^2) \tag{2}$$

$$\tau/\chi = B/R_0 = \tan\sigma. \tag{3}$$

The unit vectors along the tangent t, the principal normal n, and the binormal b of a point $0'$ on the helix constitute the Serret–Frenet frame $(0' - \boldsymbol{a}_t, \boldsymbol{a}_n, \boldsymbol{a}_b)$, which is nonorthogonal for points off the curve due to the effect of the torsion [4]. Tang has conceived a derived system which rotates with a rotation rate $-\tau$ with respect to the Serret–Frenet frame [5]. Denote the new system by vectors $\boldsymbol{a}_s, \boldsymbol{a}_m, \boldsymbol{a}_p$, and let the angle between $\boldsymbol{a}_n$ and $\boldsymbol{a}_m$ be ψ as

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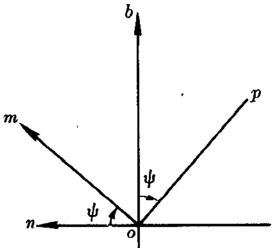

Fig. 2. The Serret–Frenet frame and Tang's coordinate system.

shown in Fig. 2, such that

$$\boldsymbol{a}_s = \boldsymbol{a}_t$$
$$\boldsymbol{a}_m = \boldsymbol{a}_n \cos\psi + \boldsymbol{a}_b \sin\psi$$
$$\boldsymbol{a}_p = -\boldsymbol{a}_n \sin\psi + \boldsymbol{a}_p \cos\psi \tag{4}$$

where

$$\frac{d\psi(s)}{ds} = -\tau. \tag{5}$$

For the helix, we have

$$\psi = -Bs/\left(R_0^2 + B^2\right). \tag{6}$$

Tang proved that the new system is orthogonal with its metric coefficients

$$h_m = h_p = 1 \quad h_s = 1 - \chi(m\cos\psi - p\sin\psi). \tag{7}$$

By changing from (m, p) to polar coordinates (r, θ)

$$m = r\cos\theta \quad p = r\sin\theta \tag{8}$$

the corresponding metric coefficients become

$$h_r = 1 \quad h_\theta = r \quad h_s = 1 - \chi r\cos(\psi + \theta). \tag{9}$$

In Tang's coordinate system, the electric- and magnetic-field vectors $\boldsymbol{E}$ and $\boldsymbol{H}$ can be written in the forms

$$\boldsymbol{E} = E_m \boldsymbol{a}_m + E_p \boldsymbol{a}_p + E_s \boldsymbol{a}_s$$
$$\boldsymbol{H} = H_m \boldsymbol{a}_m + H_p \boldsymbol{a}_p + H_s \boldsymbol{a}_s. \tag{10}$$

Now we investigate the electric-field vector wave equation

$$\left(\nabla^2 + n^2 k_0^2\right)\boldsymbol{E} = 0 \tag{11}$$

where n is the refractive index and k_0 the wavenumber in free space. Using the vector identity

$$\nabla^2 \boldsymbol{E} = \nabla(\nabla \cdot \boldsymbol{E}) - \nabla \times (\nabla \times \boldsymbol{E}) \tag{12}$$

(11) can be rewritten in the scalar-field form. After laborious but straightforward calculations, the transverse electric-field equations

$$\left[\left(\nabla^2 + n^2 k_0^2\right)\boldsymbol{E}\right]_m = 0 \quad \left[\left(\nabla^2 + n^2 k_0^2\right)\boldsymbol{E}\right]_p = 0 \tag{13}$$

become

$$\begin{aligned}
&\left[\left(\partial^2/\partial m^2 + \partial^2/\partial p^2 + h_s^{-2}\partial^2/\partial s^2 + n^2 k_0^2\right)\right.\\
&\quad + \chi\left(-\cos\psi\,\partial/\partial m + \sin\psi\,\partial/\partial p\right)h_s^{-1}\\
&\quad + \chi^2\left(-\cos^2\psi h_s^{-2} + \tan\sigma(m\sin\psi + p\cos\psi)\right.\\
&\quad \left.\left.\cdot h_s^{-3}\partial/\partial s\right)\right] E_m + \chi^2 \sin 2\psi h_s^{-2} E_p/2\\
&\quad + \left[2\chi h_s^{-2}\cos\psi\,\partial/\partial s + \chi^2\tan\sigma h_s^{-2}\sin\psi\right.\\
&\quad \left. + \chi^3 \tan\sigma h_s^{-3}(m\sin\psi + p\cos\psi)\cos\psi\right] E_s = 0
\end{aligned} \tag{14}$$

$$\begin{aligned}
&\left[\left(\partial^2/\partial m^2 + \partial^2/\partial p^2 + h_s^{-2}\partial^2/\partial s^2 + n^2 k_0^2\right)\right.\\
&\quad + \chi\left(-\cos\psi\,\partial/\partial m + \sin\psi\,\partial/\partial p\right)h_s^{-1}\\
&\quad + \chi^2\left(-\sin^2\psi h_s^{-2} + \tan\sigma(m\sin\psi + p\cos\psi)\right.\\
&\quad \left.\left.\cdot h_s^{-3}\partial/\partial s\right)\right] E_p + \chi^2 \sin 2\psi h_s^{-2} E_m/2\\
&\quad + \left[-2\chi h_s^{-2}\sin\psi\,\partial/\partial s + \chi^2\tan\sigma h_s^{-2}\cos\psi\right.\\
&\quad \left. - \chi^3 \tan\sigma h_s^{-3}(m\sin\psi + p\cos\psi)\sin\psi\right] E_s = 0.
\end{aligned} \tag{15}$$

Under the basic assumption

$$R_0 \gg a \quad \chi a \ll 1 \quad \tau a \ll 1 \tag{16}$$

it is obvious that the transverse coupling between E_m and E_p is of order $(\chi a)^2$, and the effect of coupling will not accumulate along the propagation distance s since the coupling coefficient is $[\sin 2\psi/2h_s](\chi a)^2$.

Assume that the fibers are weakly guided, i.e., $\Delta \ll 1$, the longitudinal electric field E_s is of order $\Delta^{1/2}$ of the transverse electric field [6] with Δ equal to $(n_1^2 - n_2^2)/2n_1^2$. The zero-order approximation equations of (13) and (14) will be

$$\left[\nabla_t^2 + n^2 k_0^2 - \beta_0^2\right] E_{0m} = 0 \tag{17}$$

$$\left[\nabla_t^2 + n^2 k_0^2 - \beta_0^2\right] E_{0p} = 0 \tag{18}$$

where E_{0m} and E_{0p} are the zero-order field components of E_m and E_p, while β_0 is the zero-order propagation constant, ∇_t^2 is the transverse Laplacian

$$\nabla_t^2 = \left[\partial^2/\partial\rho^2 + \partial/\rho\,\partial\rho + \partial^2/\rho^2\,\partial\theta^2\right]/a^2 \tag{19}$$

with the normalized radial coordinate

$$\rho = r/a. \tag{20}$$

The solutions of (17) and (18) with the boundary condition of the optical fiber are the same as that of straight fiber. They tend to be linear polarized waves with fixed orientation of polarization with respect to Tang's coordinate system, and the field amplitude distribution agrees with that of straight optical fibers. This conclusion has been verified by the experimental results of Papp and Harms [1]. In their experiment, a linear polarized light was launched into a helically-wound liquid-core optical fiber with its pitch angle σ equal to $\pi/4$. The polarized angle of the output wave at a distance one turn from the input end was found to be shifted with respect to the Serret–Frenet frame by an angle of $-254°$, which is consistent with (6). In fact, for $\sigma = \pi/4$, $B = R_0$, and $s = 2^{3/2}\pi R_0$, $\psi = -Bs/(R_0^2 + B^2) = -2^{1/2}\pi = -254^0$.

III. The Modal Assumption and the Perturbation Analysis of the Field and Propagation Constants

The above analysis shows that the field in a helically-wound optical fiber has a tendency to keep its propagation orientation with respect to Tang's coordinate system. So, for further analysis of the field by a perturbation method, a proper modal assumption should be chosen on the basis of Tang's coordinate system, i.e., all the subsequent analyses are based on the Maxwell equations in Tang's coordinate system, while the field components of the scalar wave equations should be the components in that system.

In using the perturbation analysis, the first-order field deformation causes the second-order propagation constant correction, which is similar to the case of the birefringence induced by the curved waveguide geometry in pure bending ($\tau = 0$) optical fiber [7]. This effect will accumulate along the propagation distance s. The correct perturbation analysis depends on the pertinent modal assumption, which is determined by the boundary conditions of the field equations. In the case of pure-bending optical fibers, the two orthogonal linearly polarized waves of the fundamental mode are parallel to $\boldsymbol{a}_n$ and $\boldsymbol{a}_b$. When τ is not equal to zero, the unit vectors $\boldsymbol{a}_m$ and $\boldsymbol{a}_p$ rotate with a rotation rate $-\tau$ with respect to $\boldsymbol{a}_n$ and $\boldsymbol{a}_b$. We can define the propagation constants β_n and β_b of the two linearly polarized modes with field components E_m and E_p at those points along the fiber where $\boldsymbol{a}_m$ and $\boldsymbol{a}_p$ coincide with $\boldsymbol{a}_n$ and $\boldsymbol{a}_b$, respectively. In the general case, i.e., on other points where $\boldsymbol{a}_m$ and $\boldsymbol{a}_p$ do not coincide with $\boldsymbol{a}_n$ and $\boldsymbol{a}_b$, the field must be split into the two states of linear polarization with their orientations parallel to $\boldsymbol{a}_n$ and $\boldsymbol{a}_b$. They propagate with two different propagation constants β_n and β_b.

Assume E_m^n and E_p^b are the distribution functions of the fundamental mode fields along the direction $\boldsymbol{a}_m$ and $\boldsymbol{a}_p$ at those points where $\boldsymbol{a}_m$ and $\boldsymbol{a}_p$ coincide with $\boldsymbol{a}_n$ and $\boldsymbol{a}_b$, respectively; the field components can then be written in the form

$$E_m^n \exp(-j\beta_n s) = (E_{0m}^n + \epsilon E_{1m}^n + \cdots)\exp(-j\beta_n s) \tag{21}$$

$$E_p^b \exp(-j\beta_b s) = (E_{0p}^b + \epsilon E_{1p}^b + \cdots)\exp(-j\beta_b s) \tag{22}$$

$$\beta_n^2 = \beta_0^2[1 + \beta_{2n}\epsilon^2 + \cdots] \tag{23}$$

$$\beta_b^2 = \beta_0^2[1 + \beta_{2b}\epsilon^2 + \cdots] \tag{24}$$

with

$$\epsilon = 2\beta_0^2 a^2 \chi a. \tag{25}$$

The longitudinal components of the electric field have the form $E_s^n \exp(-j\beta_n s)$ and $E_s^b \exp(-j\beta_b s)$. The modal assumptions (21)–(24) imply that the orientation of E_m^n and E_p^b are fixed with respect to $\boldsymbol{a}_m$ and $\boldsymbol{a}_p$ with ψ equal to $2k\pi$ ($k = 0, \pm 1, \pm 2, \cdots$). From (14) and (15), E_m^n and E_p^b should satisfy the following equations:

$$a^2(\nabla_t^2 + n^2k_0^2 - \beta_n^2/h_s^2)E_m^n = \chi a[(\cos\theta\partial/\partial\rho - \sin\theta\partial/\rho\,\partial\theta)E_m^n/h_s + 2j\beta_0 a E_s^n/h_s^2] + 0[(\chi a)^2] \tag{26}$$

$$a^2(\nabla_t^2 + n^2k_0^2 - \beta_0^2/h_s^2)E_p^b = \chi a[(\cos\theta\partial/\partial\rho - \sin\theta\partial/\rho\,\partial\theta)E_p^b/h_s] + 0[(\chi a)^2]. \tag{27}$$

Substituting (21)–(24) into (26) and (27) and equating the power of ϵ, two sets of equations for the term E_{km}^n and E_{kp}^b can be obtained. The zero-order equations are the same as (17) and (18), and the first-order equations are

$$a^2[\nabla_t^2 + n^2k_0^2 - \beta_0^2]E_{1m}^n = \rho E_{0m}\cos\theta + (\cos\theta\,\partial E_{0m}/\partial\rho - \sin\theta\,\partial E_{0m}/\rho\,\partial\theta + 2j\beta_0 a E_{0s})/2\beta_0^2 a^2 \tag{28}$$

$$a^2[\nabla_t^2 + n^2k_0^2 - \beta_0^2]E_{1p}^b = \rho E_{0p}\cos\theta + (\cos\theta\,\partial E_{0p}/\partial\rho - \sin\theta\,\partial E_{0p}/\rho\,\partial\theta)/2\beta_0^2 a^2. \tag{29}$$

E_{0s} on the left side of (28) can be obtained through the Maxwell equations in Tang's coordinate system. Here we omit the upper suffixes n and b of the zero-order field functions as they are not restricted by (21)–(24). For simplification, we consider only optical fibers with the step refractive index

$$n = \begin{cases} n_1, & (\rho < 1) \\ n_2, & (\rho > 1) \end{cases}. \tag{30}$$

Applying the boundary condition of the weakly-guided optical wave, the normal parameters are determined

$$u = a(n_1^2k_0^2 - \beta_0^2)^{1/2} \quad w = a(\beta_0^2 - n_2^2k_0^2)^{1/2}. \tag{31}$$

From (17) and (18), the zero-order field solutions are

$$E_{0m}^{(m)} = E_{0p}^{(p)} = \begin{cases} J_0(u\rho)/J_0(u), & (\rho < 1) \\ K_0(w\rho)/K_0(w), & (\rho > 1) \end{cases} \tag{32}$$

$$E_{0m}^{(p)} = E_{0p}^{(m)} = 0. \tag{33}$$

From (28) and (29), we obtain the first-order field solutions

$$E_{1m}^{n(m)} = \begin{cases} [-(-\rho^2/4u + C_1)J_1(u\rho) + 3\rho J_0(u\rho)/4\beta_0^2 a^2] \cdot\cos\theta/J_0(u), & (\rho < 1) \\ [-(\rho^2/4w^2 + C_{1e})K_1(u\rho) + 3\rho K_0(u\rho)/4\beta_0^2 a^2] \cdot\cos\theta/K_0(w), & (\rho > 1) \end{cases} \tag{34}$$

$$E_{1p}^{b(p)} = \begin{cases} [-(-\rho^2/4u + C_1)J_1(u\rho) + \rho J_0(u\rho)/4\beta_0^2 a^2] \cdot\cos\theta/J_0(u), & (\rho < 1) \\ [-(-\rho^2/4w^2 + C_{1e})K_1(u\rho) + \rho K_0(u\rho)/4\beta_0^2 a^2] \cdot\cos\theta/K_0(w), & (\rho > 1) \end{cases} \tag{35}$$

$$E_{1m}^{n(p)} = E_{1p}^{b(m)} = 0 \tag{36}$$

with c_1 and c_{1e} given by the boundary condition

$$c_1 = K_2(w)/4uK_0(w) \quad c_{1e} = J_2(u)/4wJ_0(u). \tag{37}$$

The upper suffixes (m) and (p) correspond to the linear polarized fundamental modes $\mathrm{HE}_{11}^{(m)}$ and $\mathrm{HE}_{11}^{(p)}$.

Assume A_∞ is a circle with its origin $0'$ at the center of the cross section and a large radius (e.g., $r = 100a$) in the cross-sectional plane. Carrying out the integration

$$\int_{A_\infty} \left[E_m^{n(m)} \times (18) - E_{0m} \times (26)\right] dA$$

and

$$\int_{A_\infty} \left[E_p^{b(p)} \times (17) - E_{0p} \times (27)\right] dA$$

β_{2n} and β_{2b} can be obtained after using Green's integral theorem. Finally, we yield [7]

$$\begin{aligned}\Delta\beta = \beta_b - \beta_n &\doteq \beta_0^2(\chi a)^2(\beta_{2b} - \beta_{2n})/2\beta_0 \\ &\doteq (\chi a)^2\beta_0\left[1/6 + (u^2 - w^2)/3u^2w^2 + J_0(u)/3uJ_1(u)\right].\end{aligned} \tag{38}$$

This is the birefringence due to the waveguide geometry effect. When stress-induced birefringence is considered, $\Delta\beta$ can be calculated according to the conventional formula.

Now the electric-field components E_m and E_p can be investigated in the general case. The analysis is homologous to the method used in analyzing the twisted linear birefringent fibers.

Two coordinate systems are used to discuss twisted linear birefringent fibers. The first is a fixed orthogonal system x, y, z with the corresponding unit vectors being $\boldsymbol{a}_x$, $\boldsymbol{a}_y$, and $\boldsymbol{a}_z$. The Maxwell equations in this system have a simple form. The second is a rotating nonorthogonal system x', y', z with its unit vectors $\boldsymbol{a}_{x'}$ and $\boldsymbol{a}_{y'}$ parallel to the axes of the local elliptical cross section of the fiber. In the latter system, the Maxwell equations tend to be quite tedious due to the nonorthogonality of the system. The coordinates x' and y' rotate with a twist rate α with respect to the coordinates x and y along the propagation distance z [8]. The transverse field $\boldsymbol{E}_t$ at a point of distance z in the fiber can be split into two components parallel to $\boldsymbol{a}_{x'}$ and $\boldsymbol{a}_{y'}$ at that point (as shown in Fig. 3)

$$\boldsymbol{E}_t = E_x\boldsymbol{a}_x + E_y\boldsymbol{a}_y = E_{x'}\boldsymbol{a}_{x'} + E_{y'}\boldsymbol{a}_{y'} \tag{39}$$

$$\begin{bmatrix} E_{x'} \\ E_{y'} \end{bmatrix} = \begin{bmatrix} \cos\alpha z & \sin\alpha z \\ -\sin\alpha z & \cos\alpha z \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}. \tag{40}$$

These two components propagate a small distance Δz along z with the two different propagation constants of the linear birefringent fiber in the coordinate system x, y, z; then they should be transformed back to the components of the x and y system. Here, a plane-wave approximation is used because of the weakly-guiding wave.

Now two coordinate systems are used in helically-wound optical fibers. The first is Tang's orthogonal coordinate system m, p, s. The Maxwell equations and the scalar field wave equations in this coordinate system have a simple form. The second is the fixed nonorthogonal system n, b, s. The Maxwell equations in the latter system will be quite tedious. The coordinates n and b rotate with a rotation rate τ with respect to the coordinates m and p along the propagation distance s. The transverse field $\boldsymbol{E}_t$ at a point of distance s in the fiber can be split into two components parallel to $\boldsymbol{a}_n$ and $\boldsymbol{a}_b$ at that point

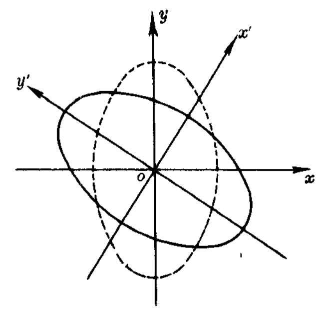

Fig. 3. Fixed and rotated coordinate systems of a twisted optical fiber.

$$\boldsymbol{E}_t = A_mE_m\boldsymbol{a}_m + A_pE_p\boldsymbol{a}_p = A_nE_m^n\boldsymbol{a}_n + A_bE_p^b\boldsymbol{a}_b \tag{41}$$

$$\begin{bmatrix} A_n \\ A_b \end{bmatrix} = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} A_m \\ A_p \end{bmatrix}. \tag{42}$$

These two components propagate a distance Δs along s with the two propagation constants of the eigenpolarization modes in bending linear birefringent fiber in the Tang's coordinate system m, p, s; then they should be transformed back to the components of the m and p system. Here, an approximation of the first-order field amplitude is used. However, the second-order difference between two propagation constants has been taken into account, so we have

$$\begin{aligned}\frac{d}{ds}\begin{bmatrix} A_m \\ A_p \end{bmatrix} = -j\begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix}\begin{bmatrix} \beta_n & 0 \\ 0 & \beta_b \end{bmatrix} \\ \times \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix}\begin{bmatrix} A_m \\ A_p \end{bmatrix}.\end{aligned} \tag{43}$$

Using (42), (43) becomes

$$\begin{aligned} dA_n/ds &= -j\beta_nA_n + \tau A_b \\ dA_b/ds &= -j\beta_bA_b - \tau A_n. \end{aligned} \tag{44}$$

Solutions of (44) can be obtained straightforwardly by the Laplace transform [9]

$$\begin{aligned}A_n(s) = \big[f\cos(\tau s/F^{1/2}) + F^{1/2}(jf\Delta\beta/2\tau + g) \\ \cdot\sin(\tau s/F^{1/2})\big]\exp(-j\beta_M s)\end{aligned} \tag{45}$$

$$\begin{aligned}A_b(s) = \big[g\cos(\tau s/F^{1/2}) - F^{1/2}(jg\Delta\beta/2\tau + f) \\ \cdot\sin(\tau s/F^{1/2})\big]\exp(-j\beta_M s)\end{aligned} \tag{46}$$

where

$$f = A_m\big|_{\substack{s=0\\ \psi=0}} \quad g = A_p\big|_{\substack{s=0\\ \psi=0}} \quad F = \left[1 + (\Delta\beta/2\tau)^2\right]^{-1}$$

$$\beta_M = (\beta_n + \beta_b)/2. \tag{47}$$

If $\Delta\beta \ll \tau$, the above field components represent a quasi-linear state of polarization, while its orientation rotates with a rotation rate close to $-\tau$ with respect to the Serret–Frenet frame.

The solutions (45) and (46) can be used directly to give the state of polarization (SOP) P, or the polarization ellipticity of a polarized wave at the output end of the fiber

$$P = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \cos\left\{\sin\left[\left(-\Delta\beta F^{1/2}/\tau\right)\sin\left(\tau s/F^{1/2}\right)\right.\right. \cdot \left(\sin 2\varphi \cos\left(\tau s/F^{1/2}\right) \left.\left. - F^{1/2}\cos 2\varphi \sin\left(\tau s/F^{1/2}\right)\right)\right]\right\} \tag{48}$$

where $I_{\max}$ and $I_{\min}$ are the maximum and minimum intensities passing through an analyzer whose direction can be adjusted, and φ is the input polarization angle with respect to the Serret–Frenet frame [10].

When

$$\varphi' = \varphi + \pi/2 \tag{49}$$

there is

$$P(\varphi') = P(\varphi). \tag{50}$$

It proves that P relates to the input polarization angles. When the input polarized angle φ changes, P changes periodically with a period $\pi/2$ at a fixed distance s. This is in agreement with the experimental results of [1]. As the ellipticity of the output wave is very close to 1, we call it quasi-linear polarization. In fact, from (48), it can be seen that for a small $\Delta\beta$, P approaches 1.

IV. Conclusion

A perturbation method is used to solve the Maxwell equations in Tang's coordinate system. The first-order field deformation and the second-order corrections of the propagation constants in single-mode helically-wound optical fibers are obtained. The theoretical analysis proves that the field in the fibers maintains a quasi-linear state of polarization, while the orientation of the polarization rotates with a rotation rate close to $-\tau$ with respect to the Serret–Frenet frame. The state of polarization periodically changes according to the incident polarized angles with a period $\pi/2$ at a fixed distance s. Our results have been verified by the experimental results of [1].

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